observations, *e.g.* by numerous repetitions. If photographs of the Moon among the stars were assiduously taken over long periods at numerous observatories, it does not seem impossible that we might obtain ultimately information as to the figure of the Earth comparable in value with that derived from telegraphic longitudes. For at present the latter are only made at rare intervals; and they are affected by some unknown, though small, sources of error, especially when submarine cables are employed. For instance, there is as yet no means of checking the fundamental assumption which enters into all telegraphic longitudes, that the time of transmission of an electric signal from A to B is the same as the time of return from B to A. The error of this assumption is no doubt a fraction of the whole time of transmission, but the latter may be as much as $0^s{\cdot}3$ across the Atlantic, and there is room for an error of $0^s{\cdot}05$ in the assumption of equal velocity to and fro. Good lunar observations systematically compared ought to be capable of easily checking such quantities as this.

Errors in the Moon's Tabular Longitude as affecting the comparison of the Greenwich Meridian Observations from 1750 *with Theory.* By P. H. Cowell.

(*Communicated by the Astronomer Royal.*)

I preface the account of my investigations with some extracts from the researches of various lunar theorists. I also exhibit a comparison between Airy's expression for the tabular longitude of the Moon and Hansen's. Had I realised when I made a certain analysis described in this paper that the errors of Airy's formula would produce a large systematic effect, as I endeavour to show in this paper that they do, my analysis would have taken a slightly different form. However, I now publish such results as I have obtained, and also the evidence on which I base the conclusion that the longitudes from 1750 to 1851 should be compared with a better formula than Airy's. I may also add that the bulk of the labour of this paper consisted in arranging the errors for analysis, and that this arrangement will enormously simplify the labour of correcting Airy's tabular places. The small fraction of the computations that represent the analysis after the arrangement was complete will in the near future be revised, but the provisional results are now published.

The arrangement which I refer to above as having formed the bulk of the labour is not described in this paper, nor are the details of the analysis. Complete information will be given with the corrected results.

The terms with coefficients over $0''{\cdot}3$, of which it is necessary to take account in the longitude of the Moon due to causes other

Erratum,

Referring to p. 22, paragraph 12. Mr. Wade's paper has been delayed in publication, and will appear in the next number of the *Monthly Notices*.

than the direct action of the Sun are approximately (1) terms due to planetary action, calculated by Radau (*Ann. de l'Observatoire de Paris, Mémoires*, vol. xxi.)

$$\begin{aligned}
&- 0''.860 \sin (V-E)\\
&+ 0.283 \sin 2(V-E)\\
&- 0.681 \sin (g+2\varpi+3V-5E)\\
&- 0.348 \sin (2V-3E+85^\circ)\\
&+ 14.42 \sin (A+30^\circ) \text{ where } A = 18V-16E-g\\
&+ 0.806 \sin (A+30^\circ-g)\\
&+ 0.756 \sin (A+30^\circ+g)\\
&- 0.422 \sin (2M-E+49^\circ)\\
&+ 0.646 \sin (E-J)\\
&- 0.881 \sin (g+2\varpi-2J)\\
&+ 0.316 \sin (g+2\varpi-3J+7^\circ)
\end{aligned}$$

to which may be added Newcomb's empirical alteration of a *Venus* term

$$-15''.5 \cos A(1+2e \cos g)$$

the second factor being introduced because $-15''.5 \cos A$ is a correction to the fundamental argument or mean anomaly. An effect which is just sensible is also produced by this term on the evection and variation ; (2) terms due to the figure of the Earth calculated by Hill (*Astron. Papers Amer. Eph.* vol. iii.)

$$\begin{aligned}
&+7''.67 \sin \Omega\\
&+1.039 \sin \Omega \cos g\\
&+0.391 \sin (2g+2\varpi-\Omega)\,;
\end{aligned}$$

(3) nutation, where two terms of Hansen are sensibly correct :

$$\begin{aligned}
&-17''.332 \sin \Omega\\
&-\ 1.254 \sin 2\odot
\end{aligned}$$

Now the meridian observations at Greenwich since 1750 fall into four groups, of which the first two slightly overlap.

(1) 1750–1851, tabular places according to Airy's formula.

(2) 1847–1861, tabular places according to Hansen's tables, with an error of sign corrected.

(3) 1862–1882, tabular places according to Hansen's tables unaltered.

(4) 1883 onwards, tabular places according to Hansen's tables, with an error of sign corrected and Newcomb's corrections introduced.

Before coming to the solar terms I set down here the corrections required by Airy and Hansen's tabular longitude to reduce them to the above formulæ, terms less than $0''\cdot3$ being omitted.

Corrections common to all four periods:

$$\begin{aligned}
&-0''\cdot681 \sin (g+2\varpi+3\mathrm{V}-5\mathrm{E})\\
&-0\cdot348 \sin (2\mathrm{V}-3\mathrm{E}+85^\circ)\\
&-0\cdot881 \sin (g+2\varpi-2\mathrm{J})\\
&+0\cdot316 \sin (g+2\varpi-3\mathrm{J}+7^\circ)
\end{aligned}$$

Additional corrections required in the first period :

$$\begin{aligned}
&+14''\cdot42 \sin (\mathrm{A}+30^\circ)\\
&+\ \ 1\cdot562 \sin (\mathrm{A}+30^\circ) \cos g\\
&-\ \ 0\cdot422 \sin (2\mathrm{M}-\mathrm{E}+49^\circ)\\
&-15\cdot5 \cos \mathrm{A}\ (1+2e \cos g)\\
&+\ \ 0\cdot45 \sin \text{☊}\\
&+\ \ 1\cdot039 \sin \qquad \cos g
\end{aligned}$$

It will be noticed that Airy gives a figure of the Earth term omitted by Hansen, and that in the coefficient of sin ☊ Airy makes two errors that nearly cancel, the term occurring both in figure of Earth and nutation.

Additional corrections required by all tabular longitudes based on Hansen's tables :

$$\begin{aligned}
&-0''\cdot92 \sin (\mathrm{A}+30^\circ)\\
&+0\cdot63 \cos \text{☊}\\
&+0\cdot445 \sin \text{☊} \cos g\\
&+0\cdot391 \sin (2g+2\varpi-\text{☊})
\end{aligned}$$

Further additional corrections required in the period 1847–1882 :

$$[1+2e\cos g]\,[-21''\cdot47 \sin (8\mathrm{V}-13\mathrm{E}+274^\circ\ 14') \\ -1''\cdot14-29''\cdot17t-3''\cdot76\,t^2-15''\cdot5 \cos \mathrm{A}]$$

and in the period 1862–1882 a still further correction

$$-0''\cdot62 \sin (2g-4g'+2\omega-4\omega')$$

Of the term $+0''\cdot63$ cos ☊ it may be mentioned that it is inserted to reduce Hansen's figure of the Earth terms to pure theory in accordance with Hill's calculations. Airy found $-1''\cdot06$ cos ☊ from the observations 1750–1851, and Hansen has apparently adopted the term with its coefficient reduced. It

will be seen in the latter part of this paper that a coefficient of the principal elliptic term 22639″·6 is obtained, whereas Airy has obtained 22639″·1. This paper sufficiently explains why Airy's value is wrong. I however tried to find a similar explanation of this cos ☊ term, and I have not succeeded; so that at the present moment I do not know whether the term really exists or not.

In the following table of solar terms in the Moon's longitude with coefficients of 0″·4 or more the *first* column gives a reference number; the *second* column classifies the term with reference to the discussion below in cases when Airy's coefficient appears to be 0″·4 or more in error; the *third*, *fourth*, and *fifth* columns exhibit the argument as sums of multiples of g, g', ω, ω'; D, g, g', F; D, g, ω, ω' respectively. The first method is Hansen's: its convenience is that the approximate period of the term is obvious from inspection; the second method is that used by Airy, Delaunay, Brown, and others: its convenience is that it gives the "characteristic" of the coefficient; the third method bears reference to the present paper; the *sixth* column gives the coefficient according to Newcomb's transformation of Hansen's theory (*Astron. Papers of Amer. Eph.* vol. i.); these coefficients correspond to the eccentricity of Hansen's tables, and to a solar parallax of 8″·848, which is not the parallax of Hansen's tables. Newcomb's comparison with Delaunay shows that these coefficients are subject to small errors only; Hansen's tables differ from Hansen's theory very slightly by quantities seldom exceeding 0″·3; the *seventh* column gives Airy's coefficient. Airy accepted Damoiseau's values of the arguments, but revised his coefficients. His action has been most unfortunate, as Damoiseau's coefficients were much superior to those that Airy substituted, and in particular Damoiseau's coefficient of the principal elliptic term was practically correct; the *eighth* column gives estimates of the correct value of two coefficients, and the consequent change in a third; the *ninth* column gives the correction to Airy's coefficient.

TABLE I.

Ref. No.	Class.	Argument expressed in Multiples of												Coefficient in Hansen's Theory modified by Newcomb.	Coefficient in Airy's Reductions.	Approximate true Value.	Correction required by Airy.
		g	g'	ω	ω'	D	g	g'	F	D	g	ω	ω'	"	"	"	"
1	A	1	...	...	...	0,	1,	0,	0	0,	1	...	...	+22640·15	+22641·6	+22639·6	−2·0
2	C	2	...	...	...	0,	2,	0,	0	0,	2	...	...	+ 769·06	+ 769·5	...	−0·4
3	C	3	...	...	...	0,	3,	0,	0	0,	3	...	...	+ 36·13	+ 36·7	...	−0·6
4	...	4	...	...	...	0,	4,	0,	0	0,	4	...	...	+ 1·94	+ 2·0	...	...
5	...	−3,	−1	...	...	0,	−3,	−1,	0	1,	−4,	−1,	1	+ 0·55	+ 0·4	...	...
6	C	−2,	−1	...	...	0,	−2,	−1,	0	1,	−3,	−1,	1	+ 7·67	+ 7·3	...	+0·4
7	C	−1,	−1	...	...	0,	−1,	−1,	0	1,	−2,	−1,	1	+ 109·92	+ 111·1	...	−1·2
8	B	0,	−1	...	...	0,	0,	−1,	0	1,	−1,	−1,	1	+ 669·85	+ 668·6	...	+1·2
9	...	1,	−1	...	...	0,	1,	−1,	0	1,	0,	−1,	1	+ 148·02	+ 148·1	...	...
10	...	2,	−1	...	...	0,	2,	−1,	0	1,	1,	−1,	1	+ 9·72	+ 9·6	...	...
11	...	3,	−1	...	...	0,	3,	−1,	0	1,	2,	−1,	1	+ 0·67	+ 0·4	...	...
12	...	−1,	−2	...	...	0,	−1,	−2,	0	2,	−3,	−2,	2	+ 1·18	+ 1·2	...	...
13	C	0,	−2	...	...	0,	0,	−2,	0	2,	−2,	−2,	2	+ 7·51	+ 7·9	...	−0·4
14	B	1.	−2	...	...	0,	1,	−2,	0	2,	−1,	−2,	2	+ 2·59	+ 2·1	...	+0·5
15	A	1,	0,	2,	−2	2,	−1,	2,	0	0,	1,	2,	−2	− 2·54	− 2·0	...	−0·5
16	B	0,	−1,	2,	−2	2,	−2,	1,	0	1,	−1,	1,	−1	+ 2·52	− 1·4	...	+3·9
17	...	1,	−1,	2,	−2	2,	−1,	1,	0	1,	0,	1,	−1	− 28·56	− 28·8	...	...
18	B	2,	−1,	2,	−2	2,	0,	1,	0	1,	1,	1,	−1	− 24·45	− 23·6	...	−0·8
19	...	3,	−1,	2,	−2	2,	1,	1,	0	1,	2,	1,	−1	− 2·93	− 2·9	...	...
20	...	−2,	−2,	2,	−2	2,	−4,	0,	0	2,	−4,	0,	0	+ 0·95	+ 0·9	...	...

Ref. No.	Class.	Argument expressed in Multiples of g, g', ω, ω'	D, g, g', F	D, g, ω, ω'	Coefficient in Hansen's Theory modified by Newcomb.	Coefficient in Airy's Reductions	Approximate true Value.	Correction required by Airy.
					$''$	$''$	$''$	$''$
21	C	−1, −2, 2, −2	2, −3, 0, 0	2, −3, 0, 0	+ 13·19	+ 12·8	...	+0·4
22	C	0, −2, 2, −2	2, −2, 0, 0	2, −2, 0, 0	+ 211·71	+ 212·4	...	0·7
23	B	1, −2, 2, −2	2, −1, 0, 0	2, −1, 0, 0	+ 4586·56	+ 4585·6	+ 4586·4	+0·8
24	C	2, −2, 2, −2	2, 0, 0, 0	2, 0, 0, 0	+ 2369·75	+ 2370·3	...	−0·5
25	...	3, −2, 2, −2	2, 1, 0, 0	2, 1, 0, 0	+ 191·95	+ 192·1	...	...
26	...	4, −2, 2, −2	2, 2, 0, 0	2, 2, 0, 0	+ 14·38	+ 14·1	...	...
27	...	5, −2, 2, −2	2, 3, 0, 0	2, 3, 0, 0	+ 1·06	+ 0·8	...	...
28	...	−1, −3 2, −2	2, −3, −1, 0	3, −4, −1, 1	+ 0·48	+ 0·3	...	...
29	C	0, −3, 2, −2	2, −2, −1, 0	3, −3, −1, 1	+ 8·66	+ 7·8	...	+0·9
30	C	1, −3, 2, −2	2, −1, −1, 0	3, −2, −1, 1	+ 206·46	+ 209·7	...	−3·2
31	B	2, −3, 2, −2	2, 0, −1, 0	3, −1, −1, 1	+ 165·52	+ 165·9	...	−0·4
32	C	3, −3, 2, −2	2, 1, −1, 0	3, 0, −1, 1	+ 14·60	+ 14·0	...	+0·6
33	B	4, −3, 2, −2	2, 2, −1, 0	3, 1, −1, 1	+ 1·18	+ 0·6	...	+0·6
34	...	1, −4, 2, −2	2, −1, −2, 0	4, −3, −2, 2	+ 7·44	+ 7·5	...	...
35	...	2, −4, 2, −2	2, 0, −2, 0	4, −2, −2, 2	+ 8·13	+ 7·8	...	...
36	B	3, −4, 2, −2	2, 1, −2, 0	4, −1, −2, 2	+ 0·76	+ 0·3	...	+0·5
37	C	2, 1, 2 ...	0, 0, 1, 2	−1, 3, 3, −1	+ 0·42	...	...	+0·4
38	...	... ... 2 ...	0, −2, 0, 2	0, 0, 2, 0	+ 1·09	+ 1·1	...	...
39	A	1, 0, 2 ...	0, −1, 0, 2	0, 1, 2, 0	− 39·58	− 37·2	...	−2·4
40	C	2, 0, 2 ...	0, 0, 0, 2	0, 2, 2, 0	− 411·60	− 411·0	...	−0·6
41	...	3, 0, 2 ...	0, 1, 0, 2	0, 3, 2, 0	− 45·09	− 45·2	...	...

Ref. No.	Class.	Argument expressed in Multiples of												Coefficient in Hansen's Theory modified by Newcomb.	Coefficient in Airy's Reductions.	Approximate true Value.	Correction required by Airy.
		g	g'	ω	ω'	D	g	g'	F	D	g	ω	ω'	$''$	$''$	$''$	$''$
42	...	4,	0,	2	...	0,	2,	0,	2	0,	4,	2,	0	−4·00	−4·1	...	...
43	C	−1,	3,	0,	2	−2,	−1,	1,	2	−3,	2,	3,	−1	+0·40	...	...	+0·4
44	...	0,	3,	0,	2	−2,	0,	1,	2	−3,	3,	3,	−1	−2·15	−2·3	...	...
45	...	−2,	2,	0,	2	−2,	−2,	0,	2	−2,	0,	2,	0	+0·43	+0·5	...	...
46	...	−1,	2,	0,	2	−2,	−1,	0,	2	−2,	1,	2,	0	+6·36	+6·2	...	...
47	...	0,	2,	0,	2	−2,	0,	0,	2	−2,	2,	2,	0	−55·25	−54·9	...	...
48	C	2,	2,	0,	2	−2,	2,	0,	2	−2,	4,	2,	0	+0·56	+0·2	...	+0·4
49	...	0,	1,	0,	2	−2,	0,	−1,	2	−1,	1,	1,	1	+1·55	+1·5	...	...
50	C	0,	0,	1,	−1	1,	−1,	1,	0	0,	0,	1,	−1	+1·33	+0·5	...	+0·8
51	A	1,	0,	1,	−1	1,	0,	1,	0	0,	1,	1,	−1	+18·09	+17·2	...	+0·9
52	...	2,	0,	1,	−1	1,	1,	1,	0	0,	2,	1,	−1	+1·27	+1·0	...	...
53	C	−1,	−1,	1,	−1	1,	−2,	0,	0	1,	−2,	0,	0	−1·78	−0·8	...	−1·0
54	B	0,	−1,	1,	−1	1,	−1,	0,	0	1,	−1,	0,	0	−18·70	−18·0	...	−0·7
55	C	1,	−1,	1,	−1	1,	0,	0,	0	1,	0,	0,	0	−125·43	−122 1	−124·8	−2·7
56	...	2,	−1,	1,	−1	1,	1,	0,	0	1,	1,	0,	0	−8·48	−8·2	...	...
57	...	3,	−1,	1,	−1	1,	2,	0,	0	1,	2,	0,	0	−0·59	−0·4	...	...
58	...	1,	−2,	1,	−1	1,	0,	−1,	0	2,	−1,	−1,	1	−0·60	−0·4	...	...
59	C	1,	−3,	3,	−3	3,	−2,	0,	0	3,	−2,	0,	0	−1·22	−0·6	...	−0·6
60	...	2,	−3,	3,	−3	3,	−1,	0,	0	3,	−1,	0,	0	−3·23	−3·0	...	...
61	C	3,	−3,	3,	−3	3,	0,	0,	0	3,	0,	0,	0	+0·41	+0·9	...	−0·5

Ref. No.	Class.	Argument expressed in Multiples of: g, g′, ω, ω′	D, g, g′, F	D, g, ω, ω′	Coefficient in Hansen's Theory modified by Newcomb.	Coefficient in Airy's Reductions.	Approximate true Value.	Correction required by Airy.
					″	″	″	″
62	...	1, 1, 1, 1	1, 0, 0, 2	−1, 2, 2, 0	+ 0·55	+ 0·7	...	...
63	...	3, −3, 4, −4	4, −1, 1, 0	3, 0, 1, −1	− 0·64	− 0·9	...	...
64	...	2, −3, 4, −4	4, −2, 1, 0	3, −1, 1, −1	− 0·36	− 0·5	...	...
65	C	1, −4, 4, −4	4, −3, 0, 0	4, −3, 0, 0	+ 1·18	+ 0·5	...	+0·7
66	C	2, −4, 4, −4	4, −2, 0, 0	4, −2, 0, 0	+ 30·78	+ 34·5	...	−3·7
67	B	3, −4, 4, −4	4, −1, 0, 0	4, −1, 0, 0	+ 38·43	+ 38·0	...	+0·4
68	C	4, −4, 4, −4	4, 0, 0, 0	4, 0, 0, 0	+ 13·90	+ 14·4	...	−0·5
69	B	5, −4, 4, −4	4, 1, 0, 0	4, 1, 0, 0	+ 1·98	+ 0·9	...	+1·1
70	C	2, −5, 4, −4	4, −2, −1, 0	5, −3, −1, 1	+ 2·75	+ 1·2	...	+1·5
71	C	3, −5, 4, −4	4, −1, −1, 0	5, −2, −1, 1	+ 4·41	+ 3·8	...	+0·6
72	B	4, −5, 4, −4	4, 0, −1, 0	5, −1, −1, 1	+ 1·89	+ 1·2	...	+0·7
73	C	2, −2, 4, −2	2, −2, 0, 2	2, 0, 2, 0	− 0·54	− 0·9	...	+0·4
74	...	3, −2, 4, −2	2, −1, 0, 2	2, 1, 2, 0	− 9·37	− 9·4	...	...
75	C	4, −2, 4, −2	2, 0, 0, 2	2, 2, 2, 0	− 5·74	− 3·4	...	−2·3
76	C	5, −2, 4, −2	2, 1, 0, 2	2, 3, 2, 0	− 0·99	− 0·6	...	−0·4
77	C	3, −3, 4, −2	2, −1, −1, 2	3, 0, 1, 1	− 0·43	...	...	−0·4
78	C	4, −6, 6, −6	6, −2, 0, 0	6, −2, 0, 0	+ 0·57	0·0	...	+0·6
79	B	5, −6, 6, −6	6, −1, 0, 0	6, −1, 0, 0	+ 0·40	...	...	+0·4
80	...	4, 0, 4, 0	0, 0, 0, 4	0, 4, 4, 0	+ 0·42	+ 0·4	...	...

In addition to the above erroneous coefficients, Airy, through a clerical error, altered the argument of a term with coefficient $0''\cdot4$; that is to say, he left out one term and put in another. The term that he left out has a coefficient of $0''\cdot22$ in Hansen, so that it does not much matter whether it was left out or inserted. It is not given in the above table. The term substituted has, according to Hansen, no sensible coefficient. Airy's tabular longitudes therefore require a further correction:

$$+0''\cdot4 \sin(-4D+g'+2F)$$

The argument may also be written

$$-2g+5g'-2\omega+4\omega'$$

or

$$-5D+3g+3\omega-\omega'$$

I shall now give a brief account of an investigation on which I have been employed during the last few months.

The tabular *minus* observed errors of the right ascensions from 1883 to 1898 were corrected by $+0^{s}\cdot133 \sin D$ for parallactic inequality, and analysed for the coefficients of $\cos g$ and $\sin g$. The errors were, in fact, equated to

$$c+a\cos g+b\sin g$$

and c, a, b obtained by a rigorous least-squares solution (see *Observatory Magazine* for 1903 September). The results obtained were:

α. On correcting a and b for certain planetary and figure of Earth terms, their accordance was improved in the ratio of 5 : 2.

β. A suspicion arose of a term with coefficient $0''\cdot5$ and argument differing from the mean anomaly by an angle whose mean motion is approximately that of the perigee.

γ. A correction to the coefficient of the principal elliptic term $-0''\cdot50$ was obtained.

δ. A correction of $-1''\cdot41 \div 0\cdot1098$ (twice the eccentricity) is required by Hansen's mean anomaly, as corrected by Newcomb in 1891, on the assumption that it is proper to reject Newcomb's empirical alteration of a certain *Venus* term. If this empirical alteration is retained the correction is one-third of the above amount.

With the object of getting fresh light on result β, I next analysed the longitudes 1750–1851, without, however, correcting the parallactic term, or any other term, in the tabular places. After equating the errors in turn to c, $a\cos g$, and $b\sin g$, and solving by least squares, I obtained values of c, a, b for eighty-nine periods of 400 lunar days into which the century's observations may be divided. Four hundred lunar days were taken as the period of analysis as being approximately equal to fifteen anomalistic months and fourteen lunations. These values of a

and b were corrected for planetary terms, and also for the solar terms Nos. 15, 39, 51 in the preceding list. Newcomb's empirical term was not inserted. There remained in the corrected values of a and b large accidental errors, and also some signs of periodicity with considerable coefficients. The accidental errors I attribute to the errors of Airy's formula for the tabular longitude, which, as the preceding table shows, are considerable. The periodicity cannot be an unknown term in the Moon's motion, as it would have appeared in 1883–1898; it is not likely to be due to systematic errors of observation, for the period is an unlikely one to arise from this cause. I attribute it to the terms marked B in the foregoing table. Possibly also the small term in the observations 1883–1898 arises in a similar manner.

It will be seen that terms of class A can be corrected for, in the values of a and b, after solution. For example, take term No. 39: here Airy's tabular place requires a correction $-2''\cdot38 \sin(g+2\omega)$; during a period of analysis in which g, it will be remembered, completes fifteen revolutions, 2ω moves 136°. If its variations, $\pm 68°$ on each side of its mean value, could be ignored, the correction to a would be $-2''\cdot38 \sin 2\omega$, and to b $-2''\cdot38 \cos 2\omega$; the coefficient has, however, been reduced to $-1''\cdot87$ as an expression of the fact that the centre of gravity of an arc of a circle is not on its circumference.

The effect of terms of class C is, I believe, purely accidental. As about 120 observations were usually to be found in a period of analysis, and as a and b are determined from these observations, with weights $\cos^2 g$ and $\sin^2 g$ respectively, the accidental errors of a and b on this account should be about one-eighth of the accidental errors of the tabular place. The last column of the table suggests $4''\cdot4$ as the probable error of a tabular place, and therefore $0''\cdot55$ as the probable error of a and b for one period. The effect of terms of class B would be accidental were it not for the fact that the observations are systematically distributed with regard to D, the age of the Moon. Let us take term No. 16 as an illustration. The formula for a is

$$a = \frac{\Sigma \text{ error} \times \cos g}{\Sigma \cos^2 g}$$

hence the systematic effect of a correction

$$+3''\cdot92 \sin(\mathrm{D}-g+\omega-\omega')$$

in the tabular place will be (over and above a considerable addition to the accidental error) approximately the mean value of

$$3''\cdot92 \sin(\mathrm{D}+\omega-\omega')$$

or

$$3''\cdot92 \sin(\omega-\omega') \times \text{mean value of } \cos \mathrm{D}$$
$$+3''\cdot92 \cos(\omega-\omega') \times \text{mean value of } \sin \mathrm{D}$$

As a rough estimate, I should equate this to

$$3''\cdot 92 \sin(\omega-\omega') \times -\tfrac{4}{9}$$
$$+3''\cdot 92 \cos(\omega-\omega') \times +\tfrac{1}{10}$$

but it is clear that a very considerable periodicity may easily result.

It is not surprising, therefore, that I have not obtained as yet any additional light on the suspected periodic term in the 1883–1898 observations. In fact, the individual values of a and b are worthless, except as confirming the above conclusions, and they should be grouped eight at a time, eight periods of analysis being approximately the period of $\omega-\omega'$, the argument of the above inequality.

Turning now to term 54 in the above list, reasoning similar to the above indicates a correction to a equal to $-0''\cdot 7 \times$ mean value of $\sin D = -0''\cdot 07$ nearly, and a correction to b equal to $+0''\cdot 7 \times$ mean value of $\cos D$, or $-0''\cdot 31$ nearly. This latter quantity is introduced into the estimate of the principal elliptic coefficient made later in this paper.

The following table gives the corrected values of a, b for the means of eight periods, which are approximately equivalent to nine years. The values here given are corrected for solar terms 15, 39, 51, and also for five planetary terms (or figure of Earth) of analogous form; that is to say, the arguments differ from g by angles of very long period.

Period.	a.	b.	Period.	a.	b.
	$''$	$''$		$''$	$''$
1– 8	+0·41	+1·49	49–56	−0·61	+2·60
5–12	+0·18	+1·84	53–60	−0·19	+2·68
9–16	−0·14	+1·63	57–64	−0·15	+2·44
13–20	−0·63	+2·00	61–68	−0·26	+2·10
17–24	−0·38	+2·60	65–72	+0·09	+2·22
21–28	−0·34	+2·49	69–76	−0·28	+2·60
25–32	−0·45	+2·27	73–80	+0·24	+2·86
29–36	−0·73	+2·40	77–84	+0·77	+2·05
33–40	−0·39	+2·41	81–88	+0·19	+2·00
37–44	−0·58	+1·48			
41–48	−0·45	+1·95	Mean	−0·21	+2·22
45–52	−0·72	+2·61			

From these figures I draw the following conclusions:

ϵ. A correction $-2''\cdot 22$ is apparently required by the principal elliptic coefficient, but this requires modification by about $+0''\cdot 19$ on account of terms Nos. 54 and 56 of the first table. This gives the true coefficient as $22639''\cdot 57$, while the R.A.'s 1883–1898 gave $22639''\cdot 65$. Newcomb has given $22639''\cdot 58$.

ζ. Damoiseau's mean anomaly for 1800 requires an increase

D

of about 2″, and its mean motion is nearly correct if we ignore long-period corrections (among them Newcomb's empirical correction) and secular terms.

η. The correction a in 1891 is $+1''\cdot41$, which becomes $+2''\cdot16$ if Damoiseau's value for the mean anomaly is used. This is quite inconsistent with the above values of a, except on the assumption that there is a long-period term or secular correction required.

Newcomb's empirical *Venus* term supplies about the right amount of curvature, its values in 1750, 1800, 1850 and 1891 being

$$+0''\cdot86, \ +1''\cdot70, \ +0''\cdot58, \ -0''\cdot96$$

making the value of a fairly constant and equal to $+1''\cdot2$, implying that Damoiseau's value of the mean anomaly is too large by 11″, or that the Hansen–Newcomb value requires a correction

$$+4''\cdot6-14''\cdot0t+4''\cdot7t^2$$

reckoning t in centuries from 1800, a formula whose values are probably within 5″ of the truth from 1750 to 1900.

I have shown in the *Observatory Magazine* for 1903 November that the consequence of assuming that no long-period terms exist other than those known from gravitational theory is that the secular term in the mean anomaly must be halved.

θ. My last conclusion is that it would be well worth while to correct Airy's tabular places for most of the larger inequalities. In Table I. a list of 47 inequalities of $0''\cdot4$ or greater is given (which can be reduced to 22 by omitting inequalities of less than $0''\cdot7$, though I do not think that this is desirable). The sum of the coefficients without regard to sign is 46″, and the probable error of a tabular place can hardly be less than $4''\cdot4$. The probable error of a discordance between theory and observation is probably capable therefore of being divided by 3, or in other words imperfections in the tabular places have hitherto reduced the weight of this century of observations to one-ninth of the value they might have. At the same time it would be worth while to similarly correct the comparison with Hansen for parallactic inequality, planetary terms, and correction to the principal elliptic inequality and value of the mean anomaly, the corrections being adjusted so that the tabular places may be as nearly continuous as possible.

Finally I give here the separate values of c, a, and b for each period, the a's and b's being corrected for the effect of terms of class A, Nos. 15, 39, and 51, and planetary terms belonging to the same class.

None of the numerical conclusions of this paper are based upon these separate values, but only on the means of eight. The separate values are given here as evidence of the necessity for revising Airy's formulæ.

Period.	No. of Obs.	c	a Corrected.	b Corrected.	Period.	No. of Obs.	c	a Corrected.	b Corrected.
		″	″	″			″	″	″
1	115	+3·38	+3·49	+1·94	40	131	−6·08	+1·09	+1·02
2	141	+2·64	+0·98	+2·59	41	127	−1·04	−0·78	+1·93
3	148	+4·02	−0·76	+0·98	42	101	−2·42	−2·25	+2·20
4	128	+3·27	−1·38	+4·05	43	127	−0·99	−1·15	+2·87
5	125	+2·60	−0·39	+0·53	44	112	−1·41	−2·70	+1·04
6	125	+0·60	−0·31	−0·03	45	145	−0·75	−1·02	+1·82
7	123	+1·29	+0·24	+0·77	46	165	−0·27	+0·93	+2·19
8	152	+1·40	+1·45	+1·07	47	158	−1·26	+3·77	+1·42
9	120	+1·68	+0·64	+3·44	48	110	−1·33	−0·38	+2·18
10	72	+0·25	+2·10	+4·41	49	119	−0·05	−1·99	+4·67
11	61	+2·50	+0·54	+2·15	50	127	−0·92	−1·61	+3·70
12	91	+2·44	−2·80	+2·38	51	132	−0·78	−3·12	+3·24
13	42	−1·79	−1·57	−2·07	52	112	−1·00	−2·34	+1·65
14	115	−0·86	−0·95	−0·45	53	117	−2·95	+0·94	+1·76
15	129	+1·11	−0·13	−0·04	54	118	−0·56	+1·03	+1·44
16	123	−1·14	+1·08	+3·24	55	119	−0·86	+1·47	+0·90
17	129	−3·72	+0·49	+4·21	56	128	−1·99	+0·77	+3·47
18	121	−4·62	−1·54	+4·01	57	87	+0·07	+0·37	+5·55
19	111	−0·26	−1·38	+3·14	58	109	+1·55	−2·21	+4·47
20	82	−0·16	−1·04	+3·95	59	93	+2·66	−2·50	+3·45
21	125	−1·98	−0·60	+0·24	60	105	+1·54	−1·40	+0·38
22	100	−4·25	−0·94	−0·01	61	109	+2·77	+0·59	+0·68
23	96	−4·34	+1·22	+2·09	62	118	+2·77	−0·12	+1·10
24	142	−3·90	+0·73	+3·15	63	123	+0·64	+1·92	+1·73
25	129	−5·61	−1·50	+6·34	64	126	+0·76	+2·12	+2·20
26	63	−2·58	−1·75	+3·10	65	112	+1·02	−1·34	+4·56
27	114	−3·21	+1·83	+3·21	66	84	+1·14	−1·30	+4·64
28	101	−1·68	−1·72	+1·78	67	126	+0·74	−2·65	+1·66
29	136	−3·12	−0·59	+0·84	68	117	+0·80	−1·31	+0·24
30	110	−2·84	+0·59	−0·82	69	112	+1·53	+0·45	+0·20
31	129	−2·82	+0·35	+1·42	70	92	−0·09	+0·81	+0·28
32	124	−3·73	−0·85	+2·27	71	104	+0·68	+3·32	+1·94
33	127	−4·49	+0·09	+3·78	72	102	+2·15	+2·72	+4·26
34	123	−4·91	−1·09	+7·09	73	81	+3·95	−0·37	+3·43
35	126	−4·27	−2·65	+3·24	74	103	+0·90	−1·70	+5·24
36	105	−5·13	−1·73	+1·37	75	115	+2·25	−1·83	+3·37
37	117	−5·08	+0·89	−0·70	76	92	+1·81	−1·14	+2·11
38	111	−4·19	−0·27	+0·38	77	100	+1·09	+1·87	+1·51
39	117	−6·21	+0·51	+3·07	78	104	+1·65	+2·42	+0·85

Period.	No. of Obs.	c	a Corrected.	b Corrected.	Period.	No. of Obs.	c	a Corrected.	b Corrected.
		″	″	″			″	″	″
79	112	+1·64	+3·05	+2·24	85	125	−1·04	+0·91	−0·44
80	111	+1·20	−0·41	+4·13	86	120	−0·62	+1·10	+2·77
81	122	+0·30	+0·85	+3·46	87	136	−2·53	+0·55	+1·90
82	99	−0·40	−1·42	+3·91	88	123	−2·91	−0·25	+4·14
83	132	−1·00	−0·96	+0·86	89	123	−3·88	−2·72	+3·47
84	105	−2·69	+0·76	−0·56					

On the Large Sun-spots of 1903 *October* 4–18 *and October* 25–*November* 6, *and the Associated Magnetic Disturbances.*

(*Communicated by the Astronomer Royal.*)

As two striking instances have occurred in less than three weeks of a marked magnetic disturbance simultaneous with the appearance of a great group of spots near the centre of the Sun's disc, some particulars of both phenomena may be of interest, though there has not been time as yet to do more than measure and reduce two or three of the photographs taken of the Sun, and though the record is imperfect until the arrival of photographs from India and Mauritius to supplement the Greenwich series.

The two groups of spots in question were both in the southern hemisphere, and both very nearly in the same latitude. The first to appear was very much the larger, and may be distinguished as the Great Southern Group; whilst the other may be termed the Second or Smaller Southern Group.

The Great Southern Group was first seen on the east limb on October 4, when a regular spot of no great area came into view. The rest of the group was not completely seen until October 5, the above-mentioned leader being separated by nearly 3° of heliographic longitude from the next portion of the group. The entire length of the group as measured on October 9 was more than 17°, and its greatest breadth in heliographic latitude was 6°·7. Its total area on this day was 2080, as expressed in the usual unit—millionths of the Sun's visible hemisphere.

The heliographic latitude of the centre of the principal spot was −21°. The heliographic longitude of the same point was 205°·5. The positions of this point relative to the central meridian of the Sun, at the times when the magnetic disturbance began and ended, and when its most marked phase began and ended, were as follows :—